Alma Mater Studiorum Università di Bologna
Laurea Magistrale in Fisica del Sistema Terra
Corso: Oceanografia Costiera

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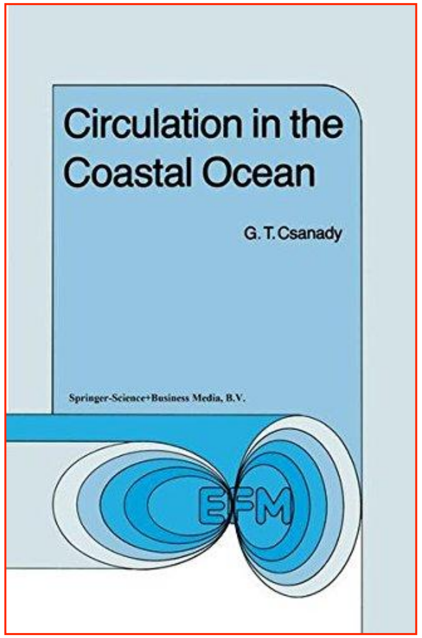
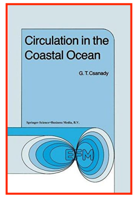
The coastal ocean response To wind (part-1)

Reference



Csanady: Chapter 2
"Inertial response to wind"
Sections 2.0, 2.1, 2.2



Wind action over the ocean

Wind blowing over the ocean surface:
Affects directly (via internal friction) a layer (in general) thinner than the whole water column

Wind effect communicated to the rest of the water column via pressure gradients that in coastal ocean are greatly influenced by the coastal constraint.

Analysis of the pressure field arising in response of wind forcing.
Issues:

- Pressure field pattern arising from a given wind forcing.
- physical mechanism leading to the establisment of such pattern
- Time needed to the pattern complete establishment
- Quantitative parameters affecting the pattern.

Faced with:
✓ Shallow water equations
✓ Homogeneous coastal ocean
✓ Constant depth (flat bottom)
✓ Straight coastlines
✓ Bottom friction neglected

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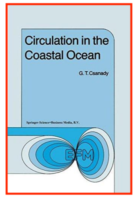
Wind “setup” (static)



The sea surface piling/depression against a leeward/windward coast due to wind action.
Consider:

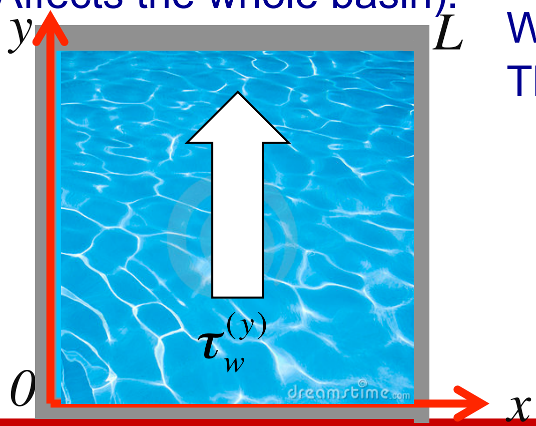
$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x} + \frac{A_v}{\rho_0}\frac{\partial^2 u}{\partial z^2} \qquad \frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y} + \frac{A_v}{\rho_0}\frac{\partial^2 v}{\partial z^2}$$

$$\frac{\partial U}{\partial t} - fV = -g(H+\eta)\frac{\partial \eta}{\partial x} + \frac{1}{\rho_0}\left(\tau_w^x - \tau_b^x\right) \qquad \frac{\partial V}{\partial t} + fU = -g(H+\eta)\frac{\partial \eta}{\partial y} + \frac{1}{\rho_0}\left(\tau_w^y - \tau_b^y\right)$$

$\tau_w^{(y)}$

$$U = \int_{-H}^{\eta} u\,dz \qquad V = \int_{-H}^{\eta} v\,dz$$

Assume a square (side length L) closed basin with constant depth H .
The basin size is assumed “sufficiently small” to have $U=V=0$ everywhere, due to the coastal constraint (i.e. the coastal constraint Affects the whole basin).



Wind blowing in one positive direction only (y).
Therefore and consequently:

$$\tau_w^{(x)} = 0 \qquad \tau_w^{(y)} > 0$$

$$\left.\frac{A_v}{\rho_0}\frac{\partial u}{\partial z}\right|_{z=\eta} = 0 \qquad \left.\frac{A_v}{\rho_0}\frac{\partial v}{\partial z}\right|_{z=\eta} = \frac{\tau_w^{(y)}}{\rho_0} = u_*^2$$

$$u_* = \sqrt{\frac{\tau_w}{\rho_0}}$$

Bottom stress negleted $\qquad \tau_b^{(x)} = \tau_b^{(y)} = 0$

Wind "setup" (static)

Under such assumptions the transport equations become:

$$\frac{\partial U}{\partial t} - fV = -gH\frac{\partial \eta}{\partial x} + \frac{1}{\rho_0}\left(\tau_w^x - \tau_b^x\right)$$

$$\frac{\partial V}{\partial t} + fU = -gH\frac{\partial \eta}{\partial y} + \frac{1}{\rho_0}\left(\tau_w^y - \tau_b^y\right)$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = -\frac{\partial \eta}{\partial t}$$

$$\Rightarrow$$

$$0 = -gH\frac{\partial \eta}{\partial x}$$

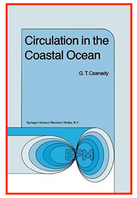
$$0 = -gH\frac{\partial \eta}{\partial y} + u_*^2$$

$$0 = \frac{\partial \eta}{\partial t}$$

$\frac{\partial \eta}{\partial y} = \frac{u_*^2}{gH}$. $u_*^2 = \tau_w^{(y)} > 0$ The sea surface slope is positive along y (toward "north")

The solution is $\boxed{\eta = \frac{u_*^2}{gH}\left(y + c_0\right)}$ where c_0 is the integration constant.

Wind "setup" (static)



$$\eta = \frac{u_*^2}{gH}\left(y + c_0\right)$$

The c_0 definition depends (in order to comply with the mass/volume conservation constraint) on the choice of the origin of the y axis.

Choosing the origin on the "southernmost" coast we have:

For $c_0 = 0$:

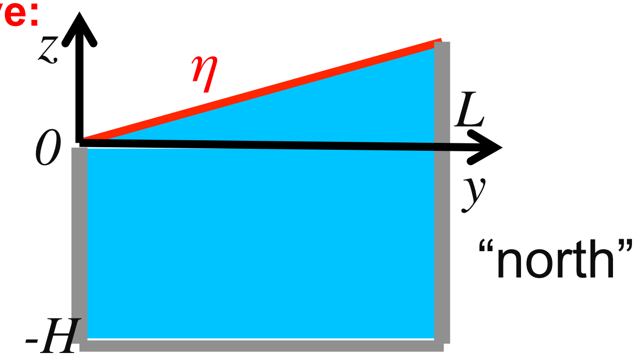
$y=0 \rightarrow \eta=0$

$y=L \rightarrow \eta = \frac{u_*^2}{gH} L$

Volume/mass is NOT conserved

$V_{rest} = L\,|H|$

$V_{setup} = L\,|H| + [L\eta_{(L)}]0.5$



For $c_0 = -L/2$ (midpoint from the 2 coasts):

$y=0 \rightarrow \eta = -\frac{u_*^2}{gH}\frac{L}{2}$

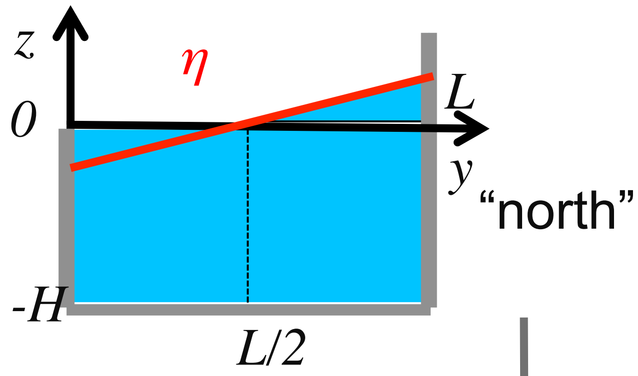
$y=L \rightarrow \eta = \frac{u_*^2}{gH}\frac{L}{2}$

Volume/mass is conserved

$V_{rest} = L\,|H|$

$V_{setup} = L\,|H| + L[\eta_{(L)} - \eta_{(0)}]0.25 =$

$= V_{rest}$



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Wind "setup" (static)

$$\boxed{\eta = \frac{u_*^2}{gH}\left(y + c_0\right)}$$

Even better............

Choosing the origin at midpoint between the two coasts we can use $c_0 = 0$:

$y=0 \rightarrow \quad \eta = 0$

$y=L/2 \rightarrow \quad \eta = \frac{u_*^2}{gH}\frac{L}{2}$

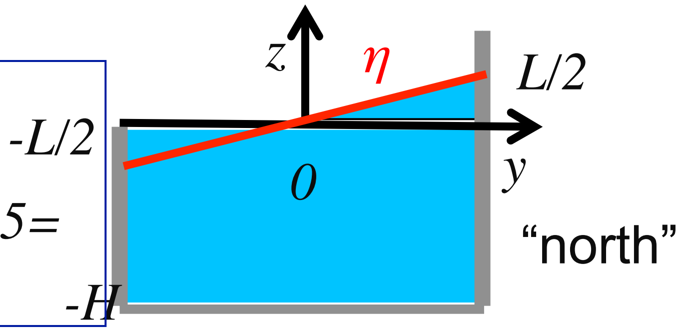
$y=-L/2 \rightarrow \quad \eta = -\frac{u_*^2}{gH}\frac{L}{2}$

Volume/mass is conserved

$V_{rest} = L\ |H|$

$V_{setup} = L\ |H| + L[\eta_{(L/2)} - \eta_{(-L/2)}]0.25 =$

$= V_{rest}$

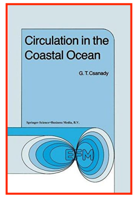


And we can safely write:

$$\boxed{\eta = \frac{u_*^2}{gH}y}$$

With this "global" solution at hand the **"steady state"** local problem (determining the interior Velocity distribution) can be solved.

Wind "setup" (static)



$$-fv = -g\frac{\partial \eta}{\partial x} + \frac{A_v}{\rho_0}\frac{\partial^2 u}{\partial z^2}$$

$$fu = -g\frac{\partial \eta}{\partial y} + \frac{A_v}{\rho_0}\frac{\partial^2 v}{\partial z^2}$$

The boundary conditions are

$$\left.\frac{A_v}{\rho_0}\frac{\partial u}{\partial z}\right|_{z=\eta} = 0 \qquad \left.\frac{A_v}{\rho_0}\frac{\partial v}{\partial z}\right|_{z=\eta} = \frac{\tau_w^{(y)}}{\rho_0} = u_*^2$$

$$\left.\frac{A_v}{\rho_0}\frac{\partial^2 u}{\partial z^2}\right|_{z=-H} = \left.\frac{A_v}{\rho_0}\frac{\partial^2 v}{\partial z^2}\right|_{z=-H} = 0$$

Wind "setup" (static)

$$-fv = -g\frac{\partial \eta}{\partial x} + \frac{A_v}{\rho_0}\frac{\partial^2 u}{\partial z^2}$$

$$fu = -g\frac{\partial \eta}{\partial y} + \frac{A_v}{\rho_0}\frac{\partial^2 v}{\partial z^2}$$

Substituting the "global" problem solution $\boxed{\eta = \frac{u_*^2}{gH}y}$ into the eq's above we get :

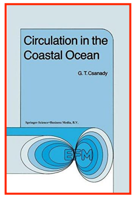
$$\boxed{-fv = \frac{A_v}{\rho_0}\frac{\partial^2 u}{\partial z^2}}$$

$$fu = -g\frac{\partial}{\partial y}\frac{u_*^2}{gH}y + \frac{A_v}{\rho_0}\frac{\partial^2 v}{\partial z^2} \Rightarrow fu = -\frac{u_*^2}{H} + \frac{A_v}{\rho_0}\frac{\partial^2 v}{\partial z^2}$$

and rearranging:

$$\boxed{f\left(u + \frac{u_*^2}{fH}\right) = \frac{A_v}{\rho_0}\frac{\partial^2 v}{\partial z^2}}$$

Wind "setup" (static)



Compare the two equations obtained with the "Ekman" Equation

$$-fv = \frac{A_v}{\rho_0}\frac{\partial^2 u}{\partial z^2}$$

$$f\left(u + \frac{u_*^2}{fH}\right) = \frac{A_v}{\rho_0}\frac{\partial^2 v}{\partial z^2}$$

$$-fv_e = \frac{A_v}{\rho_0}\frac{\partial^2 u}{\partial z^2}$$

$$fu_e = \frac{A_v}{\rho_0}\frac{\partial^2 v}{\partial z^2}$$

It can be easily seen that:

$$v = v_e$$

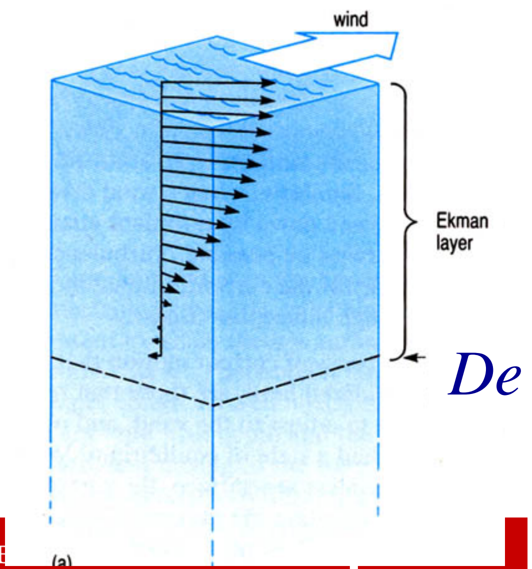
$$u = u_e - \frac{u_*^2}{fH}$$

Where u_e and v_e are the solutions of the Ekman problem (eq's 5.6 in Pinardi notes)

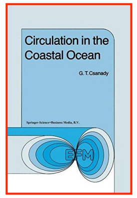
Recall also the definition of the Ekman "depth" (De) $D_e = \pi\left(\frac{2A_v}{|f|}\right)^{\frac{1}{2}}$

At $0 \le |z| \le D_e$ Ekman "spiral" development

At $|z| = D_e$ current direction opposite to surface direction and magnitude about 1/100 of surf. Value.



Wind "setup" (static)



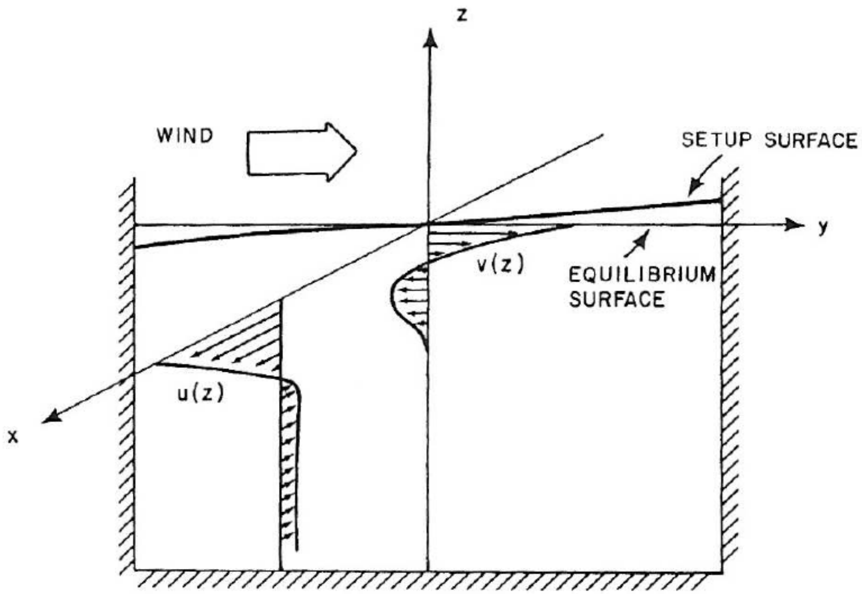
Concluding:

$$u = u_e - \frac{u_*^2}{fH}$$

$$v = v_e$$

In the surface layer:
The flow is a combination (sum) of the Ekman layer (Ekman spiral) velocity and the geostrophic velocity. Recall that:

$$g\frac{\partial \eta}{\partial y} = g\frac{\partial}{\partial y}\frac{u_*^2}{gH}$$



At depth:
For $|z| >> D_e$ (usually $|z| = 2D_e$) :

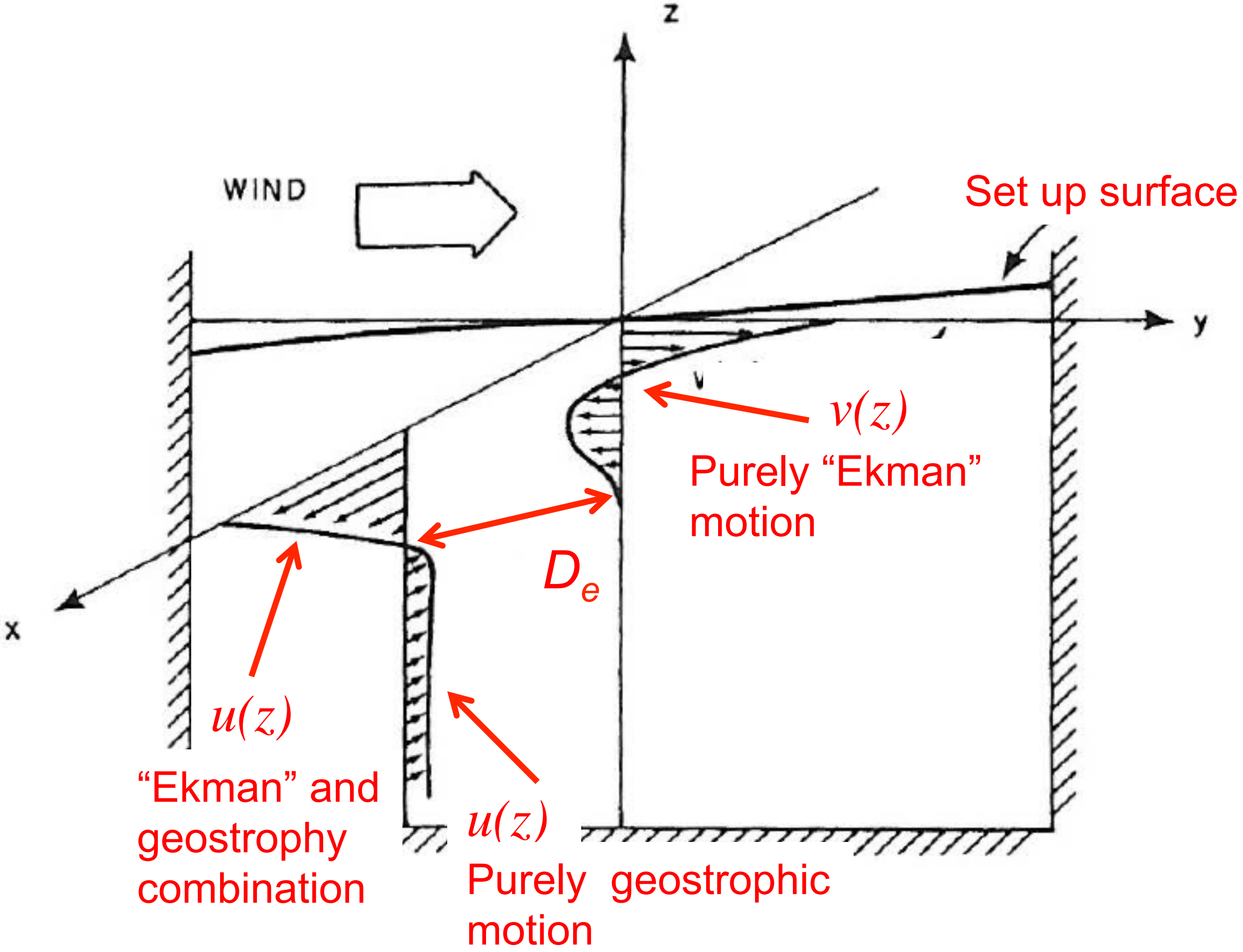
$$u = -\frac{u_*^2}{fH} \qquad v = 0$$

The flow at depth is in geostrophic balance with the pressure (sea surface) gradient

$$\frac{\partial \eta}{\partial y} = \frac{\partial}{\partial y}\frac{u_*^2}{gH}$$

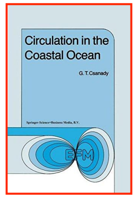
N.B.: with $\tau_w^{(x)} = 0 \quad \tau_w^{(y)} \neq 0$
we have sea surface slope along y and NOT along x

Wind "setup" (static)



Wind setup in constant depth basin and associated interior velocities. An Ekman layer at surface And geostrophic flow below.

Wind "setup" (arbitrary size basin)



Previously:

- Ekman transport $U = \dfrac{\tau_w^{(y)}}{f}$ defined by postulating vanishing pressure gradients (Pinardi notes.)
- Static wind set up $\eta = \dfrac{u_*^2}{gH} y$ defined by postulating vanishing transport (this lecture).

The above postulates justified by assuming to have the coast respectively "far" and "close" ("big" and "small" basins).

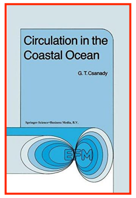
How "big"/"small" the basin should be for these limiting hypotheses to apply?

We start again with the transport equations applied to a constant depth basin.

$$\frac{\partial U}{\partial t} - fV = -c^2 \frac{\partial \eta}{\partial x} + \frac{1}{\rho_0}\left(\tau_w^x - \tau_b^x\right) \qquad \frac{\partial V}{\partial t} + fU = -c^2 \frac{\partial \eta}{\partial y} + \frac{1}{\rho_0}\left(\tau_w^y - \tau_b^y\right)$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = -\frac{\partial \eta}{\partial t}$$

Wind "setup" (arbitrary size basin)



Differentiating

$$\frac{\partial U}{\partial t} - fV = -c^2 \frac{\partial \eta}{\partial x} + \frac{1}{\rho_0}\left(\tau_w^x - \tau_b^x\right)$$ with respect to y

And $$\frac{\partial V}{\partial t} + fU = -c^2 \frac{\partial \eta}{\partial y} + \frac{1}{\rho_0}\left(\tau_w^y - \tau_b^y\right)$$ with respect to x (**as in lecture 2**)

subtracting the 2nd from the 1st and using the continuity equation:

$$\frac{\partial}{\partial t}\left(\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}\right) = f\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}\left(\tau_w^y - \tau_b^y\right) - \frac{\partial}{\partial y}\left(\tau_w^x - \tau_b^x\right)$$

We obtain a depth integrated, linearised analog of the vorticity equation:

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Wind "setup" (arbitrary size basin)

$$\frac{\partial}{\partial t}\left(\frac{\partial V}{\partial x}-\frac{\partial U}{\partial y}\right)=f\frac{\partial \eta}{\partial t}+\frac{\partial}{\partial x}\left(\tau_w^y-\tau_b^y\right)-\frac{\partial}{\partial y}\left(\tau_w^x-\tau_b^x\right)$$

Ignoring the spatial variability of the wind and bottom stress terms

$$\frac{\partial}{\partial t}\left(\frac{\partial V}{\partial x}-\frac{\partial U}{\partial y}\right)-f\frac{\partial \eta}{\partial t}=0$$

Rate of change of the Depth integrated relative Vorticity.

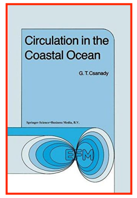
Vorticity generation Trough surface elevation Changes(squashing/stretching)

This is valid for a limited period of time. On the long run bottom friction affects inertial motion.

The time integral of the above (for $\eta=0$ at $t=0$ everywhere in the space domain) is:

$$\boxed{\frac{\partial V}{\partial x}-\frac{\partial U}{\partial y}=f\eta}$$

Wind "setup" (arbitrary size basin)



Deriving (f and c assumed constant in space):

$$\frac{\partial U}{\partial t} - fV = -c^2\frac{\partial \eta}{\partial x}$$ with respect to x $$\Rightarrow \quad \frac{\partial}{\partial x}\frac{\partial U}{\partial t} - f\frac{\partial V}{\partial x} = -c^2\frac{\partial^2 \eta}{\partial x^2}$$

$$\frac{\partial V}{\partial t} + fU = -c^2\frac{\partial \eta}{\partial y}$$ with respect to y $$\Rightarrow \quad \frac{\partial}{\partial y}\frac{\partial V}{\partial t} + f\frac{\partial U}{\partial y} = -c^2\frac{\partial^2 \eta}{\partial y^2}$$

Summing:

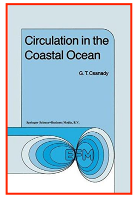
$$\frac{\partial}{\partial t}\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) - f\left(\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}\right) = -c^2\left(\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2}\right)$$

Recalling that:

$$\boxed{\begin{gathered}\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = -\frac{\partial \eta}{\partial t} \\ \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} = f\eta\end{gathered}}$$

$$\Rightarrow \quad \frac{\partial^2 \eta}{\partial t^2} + f^2\eta = c^2\left(\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2}\right)$$

Wind "setup" (arbitrary size basin)



$$\frac{\partial^2 \eta}{\partial t^2} + f^2 \eta = c^2 \left(\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} \right)$$

Posing

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Assuming steady state and rearranging

$$\nabla^2 \eta + \frac{f^2}{c^2} \eta = 0$$

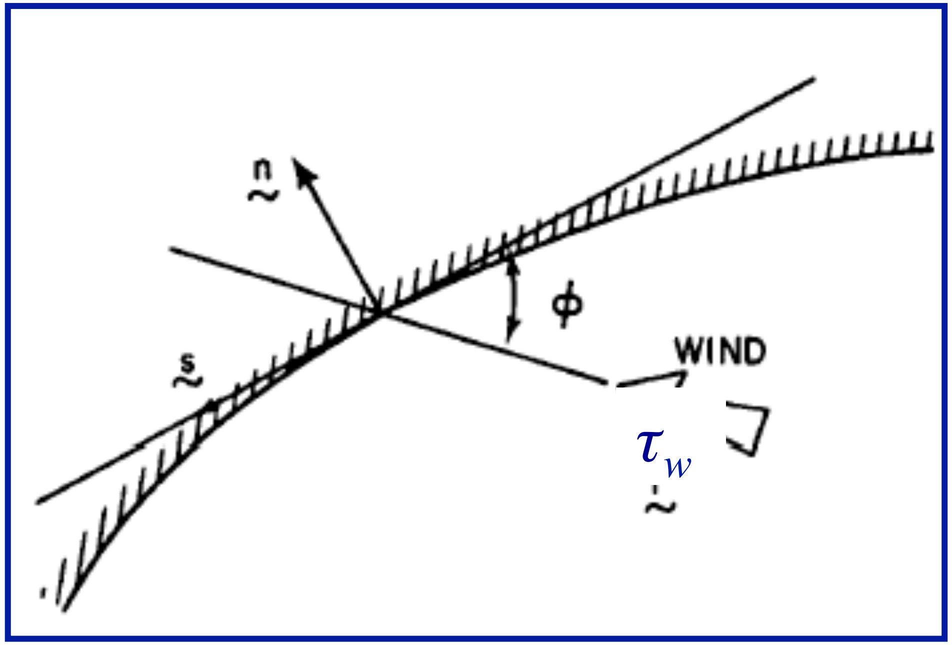
We have an equation to solve for the wind setup global problem but for the general case of "non-zero" transport everywhere.

However, normal (to the coast) transport vanishes only at the coast (boundary condition).

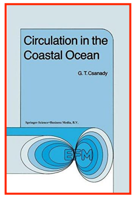
Wind "setup" (arbitrary size basin)

Normal (to the coast) transport vanishes only at the coast (boundary condition).

This may be defined expressing long- and cross-shore transport as a function of η.
Use a coordinate system (n,s) locally aligned with the normal and tangent to the coast.
($s>0$ counterclockwise, $n>0$ outward), the wind stress direction forming a variable angle with the s axis.



Wind "setup" (arbitrary size basin)



Writing the Transport equation in term of these coordinates for transport components (V_n, V_s), we can write (normal to shore transport):

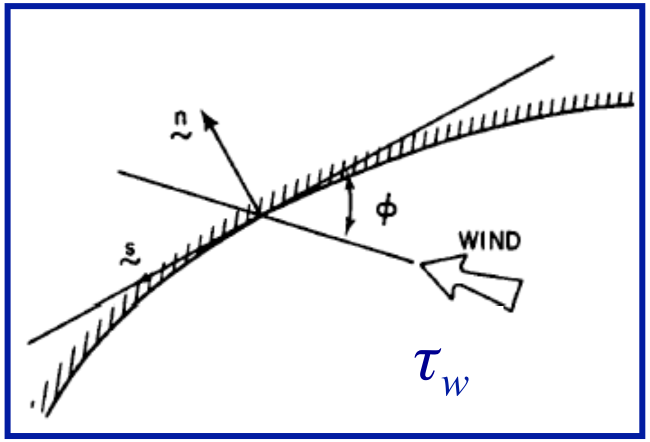
$$\frac{\partial^2 V_n}{\partial t^2} + f^2 V_n = -c^2\left(\frac{\partial^2 \eta}{\partial n \partial t} + f\frac{\partial \eta}{\partial s}\right) + f\frac{\tau_w^{(s)}}{\rho_0}$$

At steady state:

$$fV_n = -c^2\frac{\partial \eta}{\partial s} + \frac{\tau_w^{(s)}}{\rho_0}$$

At the coast $V_n=0$. then (rearranging):

$$c^2\frac{\partial \eta}{\partial s} = \frac{\tau_w^{(s)}}{\rho_0}$$

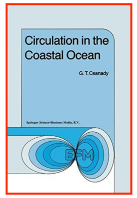


The cross-shore transport vanishes at the coast; long-shore pressure gradient and long-shore wind stress balances.

N.B.: $\tau_w^{(s)} = \tau_w \cos\phi$

$$\boxed{c^2\frac{\partial \eta}{\partial s} = \frac{\tau_w \cos\phi}{\rho_0}}$$

Wind "setup" (arbitrary size basin)



Consider a rectangular basin with side lengths $2a$ and $2b$.
Coastlines aligned with axes.

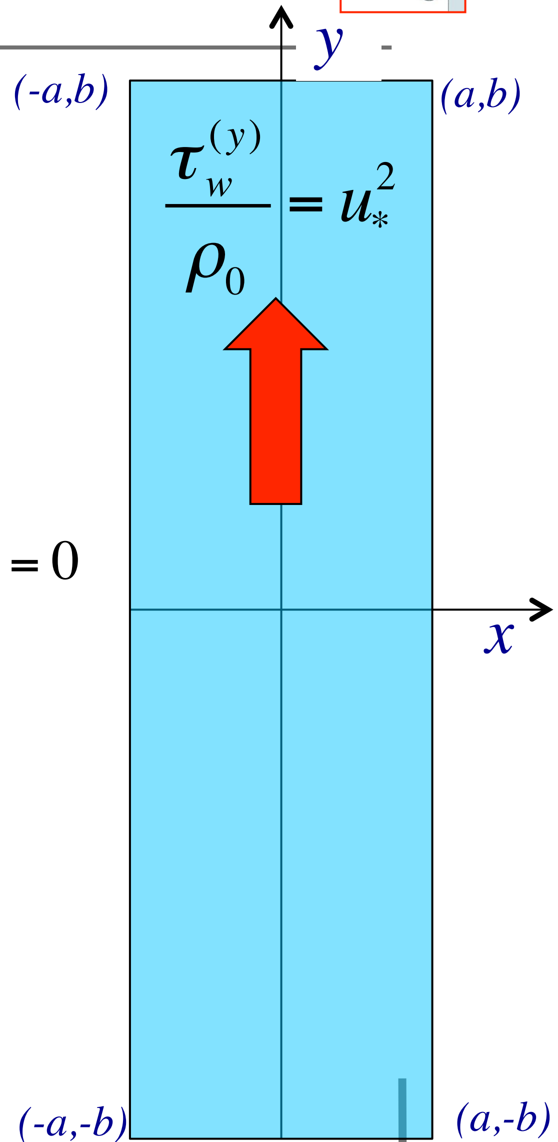
The boundary conditions are:

$$\left.\frac{\partial \eta}{\partial y}\right|_{x=\pm a} = \frac{u_*^2}{c^2} \qquad \left.\frac{\partial \eta}{\partial x}\right|_{y=\pm b} = 0$$

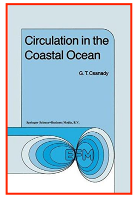
The static setup solution: $\eta = \frac{u_*^2}{gH} y$ does not satisfy: $\nabla^2 \eta - \frac{f^2}{c^2}\eta = 0$

The solution is :

$$\eta = \frac{u_*^2}{gH}\left\{ y \frac{\cosh\left(\frac{x}{R}\right)}{\cosh\left(\frac{a}{R}\right)} + \frac{4b}{R^2}\sum_{n=1}^{\infty}(-1)^{(n-1)}\frac{1}{(2n-1)l_n^2}\frac{\sinh(l_n y)}{\sinh(l_n b)}\cos k_n x \right\}$$



Wind “setup” (arbitrary size basin)



$$\eta = \frac{u_*^2}{gH}\left\{ y\frac{\cosh\left(\frac{x}{R}\right)}{\cosh\left(\frac{a}{R}\right)} + \frac{4b}{R^2}\sum_{n=1}^{\infty}(-1)^{(n-1)}\frac{1}{(2n-1)l_n^2}\frac{\sinh(l_n y)}{\sinh(l_n b)}\cos k_n x \right\}$$

$$\boxed{R = \frac{C}{f} = \frac{\sqrt{gH}}{f}} \qquad \boxed{k_n = (2n-1)(\pi/2a)} \qquad \boxed{l_n^2 = \frac{1}{R^2} - k_n}$$

***R*= External (barotropic) deformation radius [m] :**

The length scale at which rotational effects are as important as buoyancy or gravity waves effect in The flow evolution.

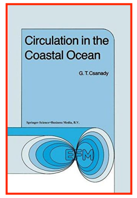
Or:

The distance that a particle or a wave travels before being significantly affected by Earth’s rotation. For

$f=10^{-4}\ s^{-1}$ $\quad H=4000\ m$ $\quad R\approx 2000\ km$

$f=10^{-4}\ s^{-1}$ $\quad H=40\ m$ $\quad R\approx 200\ km$

Wind “setup” (arbitrary size basin)



For a “small” Basin: $\dfrac{a}{R} \to 0; \dfrac{b}{R} \to 0$

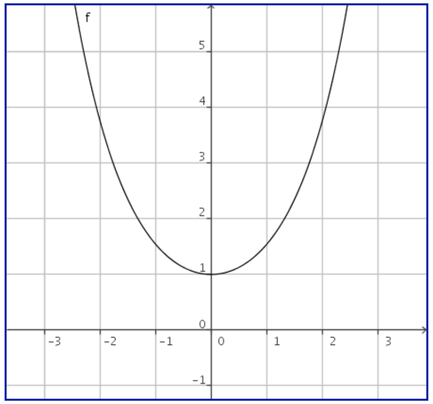
$$\eta = \frac{u_*^2}{gH}\left\{ y\frac{\cosh\left(\frac{x}{R}\right)}{\cosh\left(\frac{a}{R}\right)} + \frac{4b}{R^2}\sum_{n=1}^{\infty}(-1)^{(n-1)}\frac{1}{(2n-1)l_n^2}\frac{\sinh(l_n y)}{\sinh(l_n b)}\cos k_n x \right\}$$

$\boxed{1}$

Vanishes…

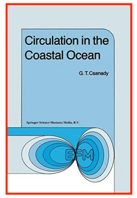
…….and the simple static setup (global) solution is recovered.

$$\eta = \frac{u_*^2}{gH} y$$



$y=cosh(x)$

Wind “setup” (arbitrary size basin)



The criterion for the approximate validity of the simple static wind setup is then:

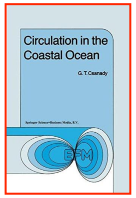
$$\boxed{\frac{(a,b)}{R} = \frac{f(a,b)}{c} = \frac{f(a,b)}{\sqrt{gH}} \rightarrow 0}$$

(a, b)= space scales of the basin.

- Small basin size
- Slow planetary rotation rates (closeness to equator)
- Large depth Basins

Approximate validity of the simple static setup solution

Wind "setup" (arbitrary size basin)



At the opposite extreme, i.e. a "large" basin with $a, b >> R$:

$$\eta = \frac{u_*^2}{gH}\left\{ y \frac{\cosh\left(\frac{x}{R}\right)}{\cosh\left(\frac{a}{R}\right)} + \frac{4b}{R^2}\sum_{n=1}^{\infty}(-1)^{(n-1)}\frac{1}{(2n-1)l_n^2}\frac{\sinh(l_n y)}{\sinh(l_n b)}\cos k_n x \right\}$$

Very large

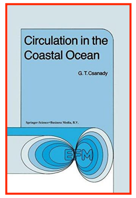
Then η assumes negligible values. However.....

except within distances of order R from boundaries: $x, y \approx R$

Very large

$$\eta = \frac{u_*^2}{gH}\left\{ y \frac{\cosh\left(\frac{x}{R}\right)}{\cosh\left(\frac{a}{R}\right)} + \frac{4b}{R^2}\sum_{n=1}^{\infty}(-1)^{(n-1)}\frac{1}{(2n-1)l_n^2}\frac{\sinh(l_n y)}{\sinh(l_n b)}\cos k_n x \right\}$$

Wind "setup" (arbitrary size basin)



Going back to the starting equation $\nabla^2\eta - \dfrac{f^2}{c^2}\eta = 0$ and noting that $\dfrac{f}{c} = R^{-1}$ we have:

$$\nabla^2\eta - \frac{f^2}{c^2}\eta = \nabla^2\eta - R^{-2}\eta = R^2\nabla^2\eta - \eta = 0$$

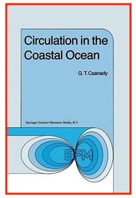
Nondimensionalising horizontal distances with the smaller side a we have:

$$\frac{R^2}{a^2}\left[\frac{\partial^2\eta}{\partial(x/a)^2} + \frac{\partial^2\eta}{\partial(y/a)^2}\right] - \eta = 0$$

That again indicates that for $a >> R,\ \eta$ becomes negligible except in boundary layers where the Horizobtal scale of variation is of order R.

Moreover…………..

Wind “setup” (arbitrary size basin)



Under steady state conditions and posing $\tau_w^{(x)} = 0$ and therefore $\tau_w^{(y)} = \rho_0 u_*^2$

And also $\tau_b^{(x)} = \tau_b^{(y)} = 0$ the transport equations:

$$\frac{\partial U}{\partial t} - fV = -c^2 \frac{\partial \eta}{\partial x} + \frac{1}{\rho_0}\left(\tau_w^x - \tau_b^x\right)$$

$$\frac{\partial V}{\partial t} + fU = -c^2 \frac{\partial \eta}{\partial y} + \frac{1}{\rho_0}\left(\tau_w^y - \tau_b^y\right)$$

Become

$$fV = c^2 \frac{\partial \eta}{\partial x}$$

$$fU = -c^2 \frac{\partial \eta}{\partial y} + \frac{\tau_w^{(y)}}{\rho_0} = -c^2 \frac{\partial \eta}{\partial y} + u_*^2$$

Under steady state conditions the horizontal transport is non-divergent:

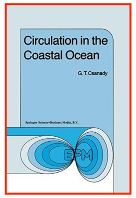
$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = -\cancel{\frac{\partial \eta}{\partial t}} \quad \Rightarrow \quad \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

And therefore can be expressed by a stream function **(see Pinardi’s notes)**

$$U = -\frac{\partial \psi}{\partial y} \qquad V = \frac{\partial \psi}{\partial x}$$

The steady state transport continuity is automatically satisfied: $\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = \frac{\partial}{\partial x}\frac{\partial \psi}{\partial y} - \frac{\partial}{\partial y}\frac{\partial \psi}{\partial x} = 0$

Wind "setup" (arbitrary size basin)



Substituting $\boxed{U = -\dfrac{\partial \psi}{\partial y}}$ $\boxed{V = \dfrac{\partial \psi}{\partial x}}$ into:

$$fV = c^2 \frac{\partial \eta}{\partial x}$$

$$fU = -c^2 \frac{\partial \eta}{\partial y} + u_*^2$$

$$\left.\begin{aligned} f\frac{\partial \psi}{\partial x} &= c^2 \frac{\partial \eta}{\partial x} \\ f\frac{\partial \psi}{\partial y} &= c^2 \frac{\partial \eta}{\partial y} - u_*^2 \end{aligned}\right\}$$

that yield $\psi = \dfrac{c^2}{f}\eta - \dfrac{u_*^2}{f} y$

With boundary condition at the shore: $\psi = 0$

Substituting η with the global solution found before and distinguishing for the "small" and "large" Basin case we get:

Wind “setup” (arbitrary size basin)

$$\boxed{\psi = \frac{c^2}{f}\eta - \frac{u_*^2}{f}y}$$

Small” basin case $\left(\frac{a}{R}, \frac{b}{R} \rightarrow 0\right)$

$\boxed{\eta = \frac{u_*^2}{gH}y}$ then $\psi \cong U \cong V \cong 0$ everywhere (the 1st simple solution found)

“Large” basin case (outside boundary layers of scale width R) $\left(\frac{a}{R}, \frac{b}{R} >> 1\right)$

$\boxed{\eta = 0}$ then $\psi \cong -\frac{u_*^2}{f}y$ and hence:

$$U = \frac{u_*^2}{f} \qquad V = 0$$

Which is the Ekman transport corresponding to a wind stress along positive y

Wind "setup" (arbitrary size basin)

Summarising:

Idealisations made: barotropic conditions, steady state,flat bottom, constant wind stress, negligible bottom stress

Large/Small basin criteria: Based on the ratio $\dfrac{a}{R}$

Solution found defining the transition from truly coastal to truly open sea conditions

However, note that for :

$$H = 30m \quad f = 10^{-4}s^{-1} \Rightarrow R = \frac{c}{f} \cong 170km$$

That is to say: In a coastal framework(shallow depth) the simple static setup solution is always a reasonable approximation
This is true for a Barotropic Ocean!!!!!!!